7.7 Transmission lines and waveguides

Transmission line relations

Loss-free transmission line equations	$\frac{\partial V}{\partial x} = -L\frac{\partial I}{\partial t}$ (7.170) $\frac{\partial I}{\partial x} = -C\frac{\partial V}{\partial t}$ (7.171)	V	potential difference across line
		I	current in line
		L	inductance per unit length
		C	capacitance per unit length
Wave equation for a lossless transmission line	$\frac{1}{LC}\frac{\partial^2 V}{\partial x^2} = \frac{\partial^2 V}{\partial t^2}$ (7.172) $\frac{1}{LC}\frac{\partial^2 I}{\partial x^2} = \frac{\partial^2 I}{\partial t^2}$ (7.173)	x	distance along line
		t	time
Characteristic impedance of lossless line	$Z_c = \sqrt{\frac{L}{C}}$ (7.174)	Z_c	characteristic impedance
Characteristic impedance of lossy line	$\boldsymbol{Z}_c = \sqrt{\frac{R+\mathrm{i}\omega L}{G+\mathrm{i}\omega C}}$ (7.175)	R	resistance per unit length of conductor
		G	conductance per unit length of insulator
		ω	angular frequency
Wave speed along a lossless line	$v_p = v_g = \frac{1}{\sqrt{LC}}$ (7.176)	v_p	phase speed
		v_g	group speed
Input impedance of a terminated lossless line	$\boldsymbol{Z}_{in} = Z_c \frac{\boldsymbol{Z}_t \cos kl - \mathrm{i} Z_c \sin kl}{Z_c \cos kl - \mathrm{i}\boldsymbol{Z}_t \sin kl}$ (7.177) $= Z_c^2/\boldsymbol{Z}_t \quad \text{if } l = \lambda/4$ (7.178)	$\boldsymbol{Z}_{in}$	(complex) input impedance
		$\boldsymbol{Z}_t$	(complex) terminating impedance
		k	wavenumber ($=2\pi/\lambda$)
Reflection coefficient from a terminated line	$\boldsymbol{r} = \frac{\boldsymbol{Z}_t - \boldsymbol{Z}_c}{\boldsymbol{Z}_t + \boldsymbol{Z}_c}$ (7.179)	l	distance from termination
		$\boldsymbol{r}$	(complex) voltage reflection coefficient
Line voltage standing wave ratio	$\text{VSWR} = \frac{1+\lvert\boldsymbol{r}\rvert}{1-\lvert\boldsymbol{r}\rvert}$ (7.180)		

Transmission line impedances[a]

Coaxial line	$Z_c = \sqrt{\frac{\mu}{4\pi^2\epsilon}}\ln\frac{b}{a} \simeq \frac{60}{\sqrt{\epsilon_r}}\ln\frac{b}{a}$ (7.181)	Z_c	characteristic impedance (Ω)
		a	radius of inner conductor
		b	radius of outer conductor
		ϵ	permittivity ($=\epsilon_0\epsilon_r$)
Open wire feeder	$Z_c = \sqrt{\frac{\mu}{\pi^2\epsilon}}\ln\frac{l}{r} \simeq \frac{120}{\sqrt{\epsilon_r}}\ln\frac{l}{r}$ (7.182)	μ	permeability ($=\mu_0\mu_r$)
		r	radius of wires
		l	distance between wires ($\gg r$)
Paired strip	$Z_c = \sqrt{\frac{\mu}{\epsilon}}\frac{d}{w} \simeq \frac{377}{\sqrt{\epsilon_r}}\frac{d}{w}$ (7.183)	d	strip separation
		w	strip width ($\gg d$)
Microstrip line	$Z_c \simeq \frac{377}{\sqrt{\epsilon_r}[(w/h)+2]}$ (7.184)	h	height above earth plane ($\ll w$)

[a]For lossless lines.

Waveguides[a]

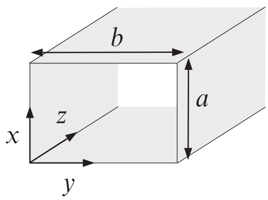
Waveguide equation	$k_{\rm g}^2=\dfrac{\omega^2}{c^2}-\dfrac{m^2\pi^2}{a^2}-\dfrac{n^2\pi^2}{b^2}$	(7.185)	$k_{\rm g}$ ω a b m,n c	wavenumber in guide angular frequency guide height guide width mode indices with respect to a and b (integers) speed of light
Guide cutoff frequency	$\nu_{\rm c}=c\sqrt{\left(\dfrac{m}{2a}\right)^2+\left(\dfrac{n}{2b}\right)^2}$	(7.186)	$\nu_{\rm c}$ $\omega_{\rm c}$	cutoff frequency $2\pi\nu_{\rm c}$
Phase velocity above cutoff	$v_{\rm p}=\dfrac{c}{\sqrt{1-(\nu_{\rm c}/\nu)^2}}$	(7.187)	$v_{\rm p}$ ν	phase velocity frequency
Group velocity above cutoff	$v_{\rm g}=c^2/v_{\rm p}=c\sqrt{1-(\nu_{\rm c}/\nu)^2}$	(7.188)	$v_{\rm g}$	group velocity
Wave impedances[b]	$Z_{\rm TM}=Z_0\sqrt{1-(\nu_{\rm c}/\nu)^2}$ $Z_{\rm TE}=Z_0/\sqrt{1-(\nu_{\rm c}/\nu)^2}$	(7.189) (7.190)	$Z_{\rm TM}$ $Z_{\rm TE}$ Z_0	wave impedance for transverse magnetic modes wave impedance for transverse electric modes impedance of free space $(=\sqrt{\mu_0/\epsilon_0})$

Field solutions for TE_{mn} modes[c]

$$B_x=\frac{\mathrm{i}k_{\rm g}c^2}{\omega_{\rm c}^2}\frac{\partial B_z}{\partial x} \qquad E_x=\frac{\mathrm{i}\omega c^2}{\omega_{\rm c}^2}\frac{\partial B_z}{\partial y}$$

$$B_y=\frac{\mathrm{i}k_{\rm g}c^2}{\omega_{\rm c}^2}\frac{\partial B_z}{\partial y} \qquad E_y=\frac{-\mathrm{i}\omega c^2}{\omega_{\rm c}^2}\frac{\partial B_z}{\partial x} \qquad (7.191)$$

$$B_z=B_0\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b} \qquad E_z=0$$



Field solutions for TM_{mn} modes[c]

$$E_x=\frac{\mathrm{i}k_{\rm g}c^2}{\omega_{\rm c}^2}\frac{\partial E_z}{\partial x} \qquad B_x=\frac{-\mathrm{i}\omega}{\omega_c^2}\frac{\partial E_z}{\partial y}$$

$$E_y=\frac{\mathrm{i}k_{\rm g}c^2}{\omega_{\rm c}^2}\frac{\partial E_z}{\partial y} \qquad B_y=\frac{\mathrm{i}\omega}{\omega_c^2}\frac{\partial E_z}{\partial x} \qquad (7.192)$$

$$E_z=E_0\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \qquad B_z=0$$

[a] Equations are for lossless waveguides with rectangular cross sections and no dielectric.
[b] The ratio of the electric field to the magnetic field strength in the xy plane.
[c] Both TE and TM modes propagate in the z direction with a further factor of $\exp[\mathrm{i}(k_{\rm g}z-\omega t)]$ on all components. B_0 and E_0 are the amplitudes of the z components of magnetic flux density and electric field respectively.